Robust mediation analysis using the R package `robmed`

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Motivation

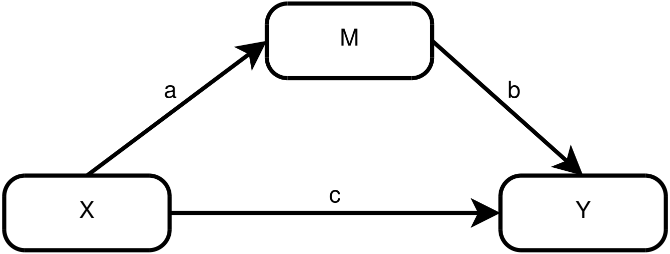
$\longrightarrow$ Effect of stimuli on behavior are mediated by various transformation processes

Simple mediation model:

X	independent variable
Y	dependent variable
M	mediator

$\longrightarrow$ Dependent variable is influenced by the independent variable through the mediator

Simple mediation model



$\longrightarrow$ X affects Y indirectly through M

Example: Task conflict (M) mediates the relationship between value diversity (X) and team commitment (Y)

Simple mediation model

Consider the following three regression models:

$$M = i_1 + aX + e_1$$
$$Y = i_2 + c'X + e_2$$
$$Y = i_3 + bM + cX + e_3$$

$\longrightarrow$ Indirect effect ab

$\longrightarrow$ Direct effect c

$\longrightarrow$ Total effect $c' = ab + c$

Simple mediation model

Consider the following three regression models:

$$M = i_1 + aX + e_1$$
$$Y = i_2 + c'X + e_2$$
$$Y = i_3 + bM + cX + e_3$$

$\longrightarrow$ Indirect effect ab

$\longrightarrow$ Direct effect c

$\longrightarrow$ Total effect $c' = ab + c$

Simple mediation model

Estimate the following two regression models:

$$M = i_M + aX + e_M$$
$$Y = i_Y + bM + cX + e_Y$$

$\longrightarrow$ Indirect effect ab

$\longrightarrow$ Direct effect c

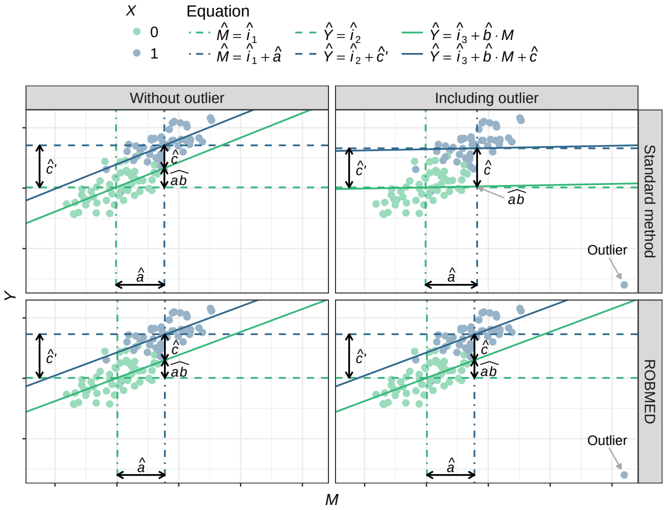
$\longrightarrow$ Total effect $c' = ab + c$

Estimation of the mediation model

- Typically, a series of ordinary least squares (OLS) regressions is used to estimate the mediation model
- Sampling distribution of the estimator $\widehat{ab}$ of the indirect effect is asymmetric

- Bootstrap is typically used to construct confidence intervals (Preacher and Hayes, 2004, 2008)

Illustration: Estimation of the mediation model



Standard bootstrap test

⟶ OLS and the bootstrap are easily distorted by deviations from the usual normality assumptions in regression

- Heavily tailed errors
- Outliers
- ...

⟶ Robust alternatives are needed to ensure reliable results in empirical research

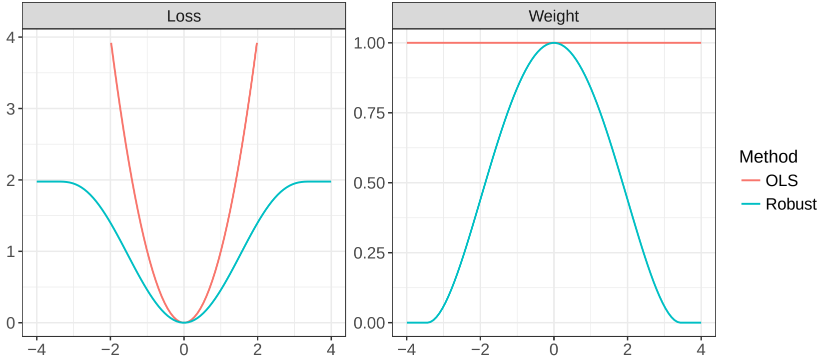
MM-estimator of regression

⟶ Replace least squares loss with a more robust loss function

⟶ Can be seen as weighted least squares estimator (WLS) with outlyingness weights derived from data

⟶ See Yohai (1987) and Salibian-Barrera and Yohai (2006)

Linear regression: Loss function and weights



Fast and robust boostrap

⟶ Not necessary to do exhaustive search for optimal weights on each bootstrap sample

⟶ On each bootstrap sample:

1. Compute WLS estimate with outlyingness weights obtained from original sample
2. Apply linear correction of estimates to account for additional uncertainty from obtaining the weights

⟶ See Salibian-Barrera and Zamar (2002) and Salibian-Barrera and Van Aelst (2008)

Robust mediation analysis

1. Estimate the mediation model via series of MM-regressions
2. Compute asymmetric confidence intervals for the indirect effect via the fast and robust bootstrap

$\longrightarrow$ Research Report: Alfons et al. (2018)

Software

⟶ R package `robmed` available on CRAN:
`https://CRAN.R-project.org/package=robmed`

⟶ R extension for SPSS under development:
`https://github.com/aalfons/ROBMED-SPSS`

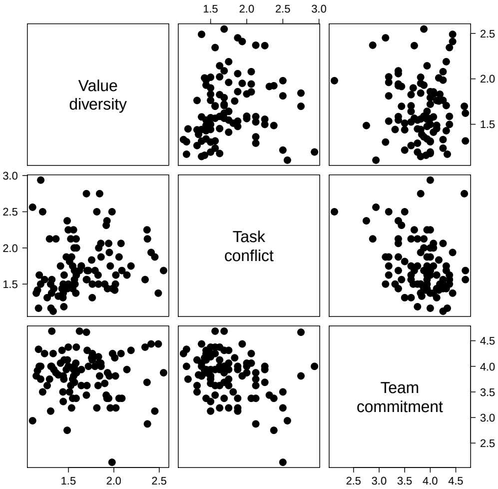
Illustrative hypotheses and data

⟶ Replicate known theory from organizational research on new data from a business strategy game played by students

Illustrative hypothesis Task conflict (M) mediates the relationship between value diversity (X) and team commitment (Y)

```
R> library("robmed")
R> data("BSG2014")
```

Empirical example



Empirical example

```
R> # seed of random number generator
R> seed <- 20150601
R>
R> # perform standard method and proposed robust method
R> set.seed(seed, sample.kind = "Rounding")  # mimic R 3.5.2
R> standard <- test_mediation(BSG2014,
+                              x = "ValueDiversity",
+                              y = "TeamCommitment",
+                              m = "TaskConflict",
+                              robust = FALSE)
R> set.seed(seed, sample.kind = "Rounding")  # mimic R 3.5.2
R> robust <- test_mediation(BSG2014,
+                            x = "ValueDiversity",
+                            y = "TeamCommitment",
+                            m = "TaskConflict")
```

Empirical example: Standard method (I)

```
R> summary(standard)
Bootstrap test for indirect effect via regression

x = ValueDiversity
y = TeamCommitment
m = TaskConflict

Sample size: 89
---
Outcome variable: TaskConflict

Coefficients:
                 Data   Boot Std. Error z value Pr(>|z|)
(Intercept)    1.5007 1.4940     0.2265   6.596 4.23e-11 ***
ValueDiversity 0.1552 0.1589     0.1266   1.255    0.209

Residual standard error: 0.3908 on 87 degrees of freedom
Multiple R-squared:  0.01857,Adjusted R-squared:  0.007289
F-statistic: 1.646 on 1 and 87 DF,  p-value: 0.2029
---
```

Empirical example: Standard method (II)

```
Outcome variable: TeamCommitment

Coefficients:
                    Data     Boot Std. Error z value Pr(>|z|)
(Intercept)      4.49846  4.50162    0.32963  13.657   <2e-16 ***
TaskConflict    -0.36412 -0.37036    0.16021  -2.312   0.0208 *
ValueDiversity -0.02088 -0.01636    0.14524  -0.113   0.9103

Residual standard error: 0.4296 on 86 degrees of freedom
Multiple R-squared:  0.1031,Adjusted R-squared:  0.08227
F-statistic: 4.944 on 2 and 86 DF,  p-value: 0.009279
---
Total effect of x on y:
                    Data     Boot Std. Error z value Pr(>|z|)
ValueDiversity -0.07738 -0.07609    0.15855   -0.48    0.631

Direct effect of x on y:
                    Data     Boot Std. Error z value Pr(>|z|)
ValueDiversity -0.02088 -0.01636    0.14524  -0.113     0.91
```

Empirical example: Standard method (III)

```
Indirect effect of x on y:
                Data     Boot   Lower  Upper
TaskConflict -0.0565 -0.05973 -0.2083 0.0251
---
Level of confidence: 95 %

Number of bootstrap replicates: 5000
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Function `p_value()` allows to extract the smallest α for which the $(1 - \alpha) \cdot 100\%$ confidence interval does not contain 0:

```
R> p_value(standard)
[1] 0.1584
```

Empirical example: ROBMED (I)

```
R> summary(robust)
Robust bootstrap test for indirect effect via regression

x = ValueDiversity
y = TeamCommitment
m = TaskConflict

Sample size: 89
---
Outcome variable: TaskConflict

Coefficients:
                Data   Boot Std. Error z value Pr(>|z|)
(Intercept)    1.1182 1.1162     0.1778   6.279 3.42e-10 ***
ValueDiversity 0.3197 0.3211     0.1071   2.998  0.00272 **

Robust residual standard error: 0.3033 on 87 degrees of freedom
Robust R-squared:  0.1181,Adjusted robust R-squared:  0.108
Robust F-statistic: 9.113 on 1 and Inf DF,  p-value: 0.002539
---
```

Empirical example: ROBMED (II)

```
Outcome variable: TeamCommitment

Coefficients:
                   Data     Boot Std. Error z value Pr(>|z|)
(Intercept)     4.33385  4.34430    0.34088  12.744   <2e-16 ***
TaskConflict   -0.33659 -0.34353    0.17761  -1.934   0.0531 .
ValueDiversity  0.06523  0.06507    0.18594   0.350   0.7264

Robust residual standard error: 0.3899 on 86 degrees of freedom
Robust R-squared:  0.08994,Adjusted robust R-squared:  0.06878
Robust F-statistic: 1.497 on 2 and Inf DF,  p-value: 0.2239
---
Total effect of x on y:
                   Data     Boot Std. Error z value Pr(>|z|)
ValueDiversity -0.04239 -0.04501    0.18671  -0.241     0.81

Direct effect of x on y:
                  Data    Boot Std. Error z value Pr(>|z|)
ValueDiversity 0.06523 0.06507    0.18594    0.35    0.726
```

Empirical example: ROBMED (III)

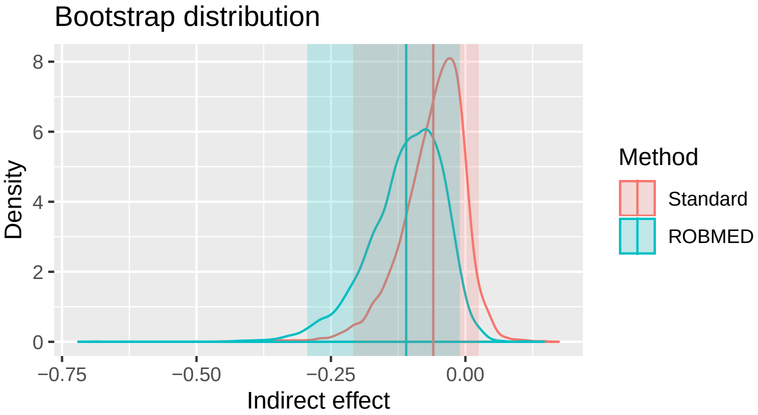
```
Indirect effect of x on y:
                Data    Boot  Lower    Upper
TaskConflict -0.1076 -0.1101 -0.294 -0.01042
---
Level of confidence: 95 %

Number of bootstrap replicates: 5000
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Function `p_value()` allows to extract the smallest α for which the $(1-\alpha)\cdot 100\%$ confidence interval does not contain 0:

```
R> p_value(robust)
[1] 0.0271
```

Empirical example

```
R> plot_mediation(list(Standard = standard, ROBMED = robust),
+                 method = "density")
```



R package `robmed`: Further details

→ The usual `coef()`, `confint()`, `plot()`, `print()` and `summary()` methods

→ Other techniques: based on winsorization (Zu and Yuan, 2010) or median regression (Yuan and MacKinnon, 2014)

→ Multiple mediators or additional covariates

Conclusions and discussion

Conclusions

- $\longrightarrow$ Standard bootstrap test for mediation analysis is easily distorted by deviations from the usual normality assumptions
- $\longrightarrow$ Fast and robust bootstrap allows for much more reliable empirical results than other methods
- $\longrightarrow$ R package `robmed` available on CRAN

Future work

- $\longrightarrow$ Binary/nominal/ordinal dependent variable or mediators
- $\longrightarrow$ Mediated moderation, moderated mediation, ...

References

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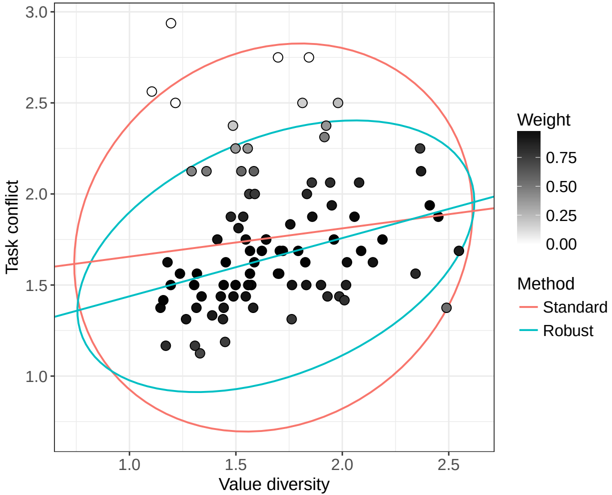
V.J. Yohai. High breakdown-point and high efficiency robust estimates for regression. **The Annals of Statistics**, 15(20):642–656, 1987.

Y. Yuan and D.P. MacKinnon. Robust mediation analysis based on median regression. **Psychological Methods**, 19(1):1–20, 2014.

J. Zu and K.-H. Yuan. Local influence and robust procedures for mediation analysis. **Multivariate Behavioral Research**, 45(1):1–44, 2010.

Empirical example: Further analysis



Technical details: MM-estimator

⟶ MM-estimator (Yohai, 1987; Salibian-Barrera and Yohai, 2006) minimizes a bounded function $\rho(r)$ of the scaled residuals:

$$\hat{\beta} = \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^{n} \rho\left(\frac{r_i(\beta)}{\hat{\sigma}}\right),$$

where $\hat{\sigma}$ is a robust scale estimate of the residuals from a highly robust initial regression estimator

⟶ Loss function ρ can be tuned for high efficiency

⟶ High robustness is inherited from initial residual scale $\hat{\sigma}$

Technical details: MM-estimator

The MM-estimate can be written as

$$\hat{\beta} = \left(\sum_{i=1}^{n} w_i \mathbf{x}_i \mathbf{x}_i^T \right)^{-1} \sum_{i=1}^{n} w_i \mathbf{x}_i y_i$$

with

$$w_i = \frac{\rho'(r_i(\beta)/\hat{\sigma})}{r_i(\beta)/\hat{\sigma}}, \qquad i = 1, ..., n$$

$\longrightarrow$ Weighted least squares estimator (WLS) with outlyingness weights derived from data